



Artificial intelligence-based expert weighted quantum picture fuzzy rough sets and recommendation system for metaverse investment decision-making priorities

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Abstract

There should be some improvements to increase the performance of Metaverse investments. However, businesses need to focus on the most important actions to provide cost effectiveness in this process. In summary, a new study is needed in which a priority analysis is made for the performance indicators of Metaverse investments. Accordingly, this study aims to evaluate the main determinants of the performance of the metaverse investments. Within this context, a novel model is created that has four different stages. The first stage is related to the prioritizing the experts with artificial intelligence-based decision-making method. Secondly, missing evaluations are estimated by expert recommendation system. Thirdly, the criteria are weighted with Quantum picture fuzzy rough sets-based (QPFR) M-Step-wise Weight Assessment Ratio Analysis (SWARA). Finally, investment decision-making priorities are ranked by QPFR VIKOR (Vlse Kriterijumska Optimizacija Kompromisno Resenje). The main contribution of this study is the integration of the artificial intelligence methodology to the fuzzy decision-making approach for the purpose of computing the weights of the decision makers. Owing to this condition, the evaluations of these people are examined according to their qualifications. This situation has a positive contribution to make more effective evaluations. Organizational effectiveness is found to be the most important factor in improving the performance of metaverse investments. Similarly, it is also identified that it is important for businesses to ensure technological improvements in the development of Metaverse investments. On the other side, the ranking results indicate that regulatory framework is the most critical alternative in this regard.

Keywords Artificial intelligence · Recommendation system · Collaborative filtering · Metaverse · Investment analysis · Fuzzy decision-making · Quantum picture fuzzy rough set

1 Introduction

Metadata is data that describes the content and structure of a data in detail. Metadata has a key role in managing and storing this data effectively. Thanks to this data, large data sets can be processed effectively. Metadata investments are projects carried out for the development of data management technologies. In this way, it is possible to both manage the data correctly and ensure its security (Lyu et al. 2024). On the other hand, metaverse investments refer to financial investments made in companies and technologies operating in the digital universe. These investments benefit businesses in many ways. Metaverse investments significantly help the development of the digital economy. This situation allows businesses to increase their profit margin. Moreover, this platform also encourages the emergence of innovative solutions in the virtual world. In other words, thanks to the development of these projects, technological progress can be achieved. In this context, it is possible to increase the trade of digital assets with new investments (Johri et al. 2024). In this way, it is possible to increase the sales volumes of businesses. On the other hand, increasing metaverse investments allows the development of many different sectors. Thus, it becomes more possible to develop country economies (Lo et al. 2024a). There are some issues that affect the performance of Metaverse investments. The financial efficiency of businesses plays an important role in the development of metaverse investments. Similarly, ensuring customer satisfaction and increasing technological sufficiency are also very necessary in this process (Maden and Yücenur 2024).

To increase the effectiveness of Metaverse investments, some improvements need to be made. This supports the increase in the benefits provided by these projects. However, these improvements also lead to increased costs. Therefore, making too many improvements causes businesses to experience some financial problems (Isabels et al. 2024). Hence, businesses need to focus on the most important actions to increase the performance of these investments. However, the number of studies in the literature identifying the most important of these factors is insufficient (Dolata and Schwabe 2023a, b; Pamucar and Biswas 2023). In summary, a new study is needed in which a priority analysis is made for the performance indicators of Metaverse investments. Accordingly, this study aims to evaluate the main determinants of the performance of the metaverse investments. Within this context, a novel model is created that has four different stages. The first stage is related to the prioritizing the experts with artificial intelligence-based decision-making method. Secondly, missing evaluations are estimated by expert recommendation system. Thirdly, the criteria are weighted with Quantum picture fuzzy rough sets (QPFR) M-Step-wise Weight Assessment Ratio Analysis (SWARA). Furthermore, investment decision-making priorities are ranked by QPFR Vlse Kriterijumska Optimizacija Kompromisno Resenje (VIKOR).

The need for a new fuzzy decision-making model for the evaluation of the performance indicators of the metaverse investments is the main motivation to generate this manuscript. Existing model generally do not consider the weights of the decision-makers. To satisfy this situation, the weights of the experts are computed in this proposed model. Another important motivation of this study is that a new study is needed to understand how to increase the performance of Metaverse investments in an efficient way. In this process, investors should make appropriate strategic decisions for the improvements of these projects. Otherwise, businesses may have to bear very high costs while trying to develop these projects because of the wrong decisions they make. Therefore, to make more accurate investment decisions, it is necessary to understand the most important factors affecting the performance of Metaverse projects. As can be seen, it is necessary to understand the

issues that most affect the performance of these investments. This is necessary for investors to make correct strategic investment decisions. Based on this need, a priority analysis for these performance indicators is carried out in this study.

Therefore, the main contributions of this manuscript are explained as follows. (i) Artificial intelligence is combined with the fuzzy decision-making approach to compute the weights of the decision makers. Owing to this condition, the evaluations of these people are examined according to their qualifications. Therefore, more effective analysis can be conducted. (ii) By the help of collaborative filtering methodology, missing evaluations can be completed. This process provides an opportunity to the decision makers not to evaluate a question when they do not have sufficient knowledge. This situation has a powerful contribution to reach more appropriate findings. (iii) Considering M-SWARA methodology helps to create causal directions among the criteria. This new methodology was generated while making some improvements to the classical SWARA approach. These improvements help to understand the impact relation map. Performance indicators of the metaverse investment can affect each other in a significant manner. Due to this situation, it is very appropriate to consider M-SWARA technique in the analysis process. (iv) The integration of Quantum theory with picture fuzzy rough sets provides some benefits. Quantum theory is mainly used in the science of physics. The main advantage of this theory is making successful future estimations. On the other side, the main problem in fuzzy decision-making methodology is managing uncertainty. To minimize this problem, Quantum theory is used with picture fuzzy rough numbers in this proposed model.

The second part focuses on literature evaluation. Proposed methodology is explained in the next section. The following part includes analysis results. Discussions and conclusions are explained in the final sections.

2 Literature review

Financial efficiency is of key importance in increasing the effectiveness of metaverse investments. Kang and Ki (2024) stated that the costs of the projects must be managed successfully. This contributes to increasing the financial performance of businesses. Otherwise, costs will increase uncontrollably, and this causes the financial profit margin of the projects to decrease. Moreover, Li et al. (2024) defined that to ensure financial performance, the financial resources of Metaverse investments must be used effectively. This helps businesses use their budgets more successfully. Cruz et al. (2024) identified that effective risk management is also of serious importance in ensuring the financial efficiency of metaverse investments. Kim and Yoo (2024) concluded that taking the right precautions against risks helps increase the financial success of projects. Therefore, the risks in metaverse investments need to be determined accurately. Zhao et al. (2024) indicated that if this analysis is not carried out comprehensively, these unmanageable risks may cause some financial problems. Similarly, businesses that have achieved financial efficiency can gain a competitive advantage through metaverse investments. This situation makes businesses more preferred by both customers and investors.

Ensuring customer satisfaction for businesses is another factor necessary for the development of metaverse investments. For these investments to be successful, the platform must be accepted by customers. Boo and Suh (2024) concluded that satisfied customers are willing to make more transactions on these platforms. Therefore, customers' expectations for these projects need to be clearly determined. In this context,

according to Shin et al. (2024), it is necessary to carry out a detailed study and clearly determine the expectations of customers in this regard. On the other hand, ensuring customer satisfaction also plays an important role in gaining new customers. Adil et al. (2024) underlined that thanks to developing technology, the opinions of customers who are satisfied with the services can be heard more easily by other people. In other words, customers who are satisfied with the services can advertise their businesses very successfully. Sylaiou et al. (2024) defined that feedback from customers regarding the services provided should be provided. This feedback provides comprehensive information about the problems in the process. Büchel and Spinler (2024) showed that by analyzing this information correctly, correct investment strategies can be offered to businesses.

Technological development should be ensured in the development of businesses' metaverse investments. Abdari et al. (2024) mentioned that transactions on the Metaverse platform involve extensive processes. Thus, technological innovation must be increased for investments in this platform to be successful. Similarly, Jaheer Mukhtar et al. (2024) identified that the use of innovative technologies helps businesses operate more effectively compared to other companies. The use of blockchain technology allows metaverse investments to perform better. Den Yeoh et al. (2023) defined that owing to this technology, it becomes easier to increase the trading of digital assets. Therefore, the development of this technology supports improving the performance of investments on this platform. Turi (2023) concluded that metaverse applications can offer more personalized experiences with artificial intelligence-style approaches. Thanks to analyzes carried out with artificial intelligence applications, it is possible to meet customer expectations more successfully. In addition, according to Besson and Gauttier (2023), necessary security measures must be taken to increase the performance of metaverse investments. This increases investors' confidence in the system. Hence, this goal must be achieved by making the necessary technological investments.

Organizational effectiveness supports businesses in successfully managing their metaverse investments. Dolata and Schwabe (2023a, b) identified that business processes need to be optimized. This situation enables costs to be reduced. Mosco (2023) denoted that experiencing disruptions in business processes creates new costs. If this situation is not controlled, it causes businesses to suffer significant financial losses. Konyalioglu (2023) discussed that metaverse projects, on the other hand, often require collaboration between different departments and teams. This issue plays a major role in increasing the effectiveness of business processes. Demir et al. (2023) demonstrated that when employing personnel, businesses should make sure that these people have strong communication skills. In addition, ensuring organizational effectiveness also enables operational risks to be handled more successfully. Banaeian Far and Hosseini Bamakan (2023) defined that owing to the departments that can work in coordination with each other, it is possible to detect risks in the process more accurately. Thus, Li and Li (2024) concluded that it can be easier to prevent possible problems by taking more effective and timely measures.

As a result of the literature review, it is possible to reach some important points. Metaverse investments have gained serious importance especially in recent years. Increasing the performance of these investments has a great impact on the development of the country's economy. Therefore, necessary actions need to be taken to increase these investments. However, these improvements create new costs for businesses. In other words, making improvements to many factors radically increases costs. Since this situation will cause financial difficulties for businesses, businesses need to

make improvements for fewer but very important factors. In this context, important factors affecting the performance of Metaverse investments need to be determined. Nonetheless, the number of the studies in this subject are not sufficient. This situation can be accepted as an important missing part in the literature. To satisfy this issue, in this study, it is aimed to examine the main determinants of the performance of the metaverse investments with a new decision-making model.

3 Methodology

Determining the most optimal investment decision-making priorities in the newly developing metaverse market becomes increasingly important for investors. For this purpose, it is necessary to determine the effective factors and determine the weights for ranking the investments. For this, in the proposed model, balanced-scorecard-based criteria are first determined and weighted using multi-SWARA. In next stage of this article, investment decisions are prioritized using VIKOR method. In proposed model, fuzzy logic is integrated into the model to take into account the uncertainty in expert opinions and the analysis process. Quantum picture fuzzy rough (QPFR) is preferred. Within the proposed model, fuzzy logic is integrated to incorporate uncertainty inherent in expert opinions and the analysis process. Additionally, experts need to be prioritized in expert opinion systems. Since giving equal importance to experts is criticized in the literature, it is recommended to prioritize experts with artificial intelligence-based k-means clustering (Romanuke 2023). In addition, obtaining incomplete expert opinions is one of the risks of the process. As a precaution, estimation for missing values is recommended.

The proposed methodology integrates various advanced techniques to address the intricate challenges of determining optimal investment decision-making priorities in the emerging metaverse market. The rationale behind utilizing these methods lies in their unique capabilities and advantages, as well as their collective ability to provide a comprehensive and robust analysis. For this purpose, firstly, the integration of quantum mechanics, picture fuzzy logic, and rough sets brings a novel dimension to the analysis. Quantum mechanics-inspired approaches, such as quantum picture fuzzy rough sets (QPFRSs), allow for the representation of complex uncertainties and ambiguities inherent in decision-making processes. By leveraging principles from quantum mechanics, such as wave functions and phase angles, QPFRSs provide a more nuanced understanding of decision criteria and expert opinions, leading to more accurate and realistic results. Picture fuzzy logic further enhances this by enabling experts to provide nuanced evaluations, including positive, negative, neutral, and refusal opinions, thereby capturing the complexity of real-world decision-making scenarios. Additionally, rough sets theory helps to minimize subjective evaluations by providing a structured framework for handling uncertainty and vagueness in decision analysis.

Secondly, AI-based expert prioritization through techniques like k-means clustering addresses the challenge of effectively utilizing expert opinions in decision-making processes. The choice of k-means clustering for prioritizing experts is justified by its ability to efficiently group experts based on similarities in their opinions. Unlike other clustering algorithms, k-means is well-suited for handling large datasets and can easily adapt to different types of data. By prioritizing experts through k-means clustering, the study ensures that more weight is given to the opinions of experts who share similar perspectives, enhancing the reliability of the analysis.

Furthermore, collaborative filtering techniques are employed to enhance the completeness and accuracy of the dataset. By leveraging the collective preferences and opinions of experts, collaborative filtering enables the completion of missing values in the dataset, ensuring that the analysis is based on a more comprehensive and representative set of data. This approach not only improves the quality of the analysis results but also helps to prevent biases and inconsistencies that may arise from incomplete data.

However, the utilization of the extended VIKOR and M-SWARA methods is motivated by their suitability for multi-criteria decision-making (MCDM) scenarios. The extension of SWARA entitled M-SWARA provides a robust framework for weighting and determining optimal investment decision-making priorities (Jafarzadeh Ghouschi and Sarvi 2023; Lo et al. 2024b), M-SWARA enables the systematic assessment and prioritization of diverse criteria that influence investment decisions (Chaurasiya and Jain 2023; Bouraima et al. 2024). VIKOR is particularly advantageous in its ability to provide compromise solutions that balance conflicting criteria, making it ideal for ranking alternatives in decision-making processes. These methods offer a nuanced approach to decision-making, considering multiple factors and expert opinions simultaneously, which sets them apart from algorithms focused solely on clustering or single-criterion decision-making.

The proposed model is summarized in Fig. 1.

Details of the steps mentioned in Fig. 1 are presented under subtitles.

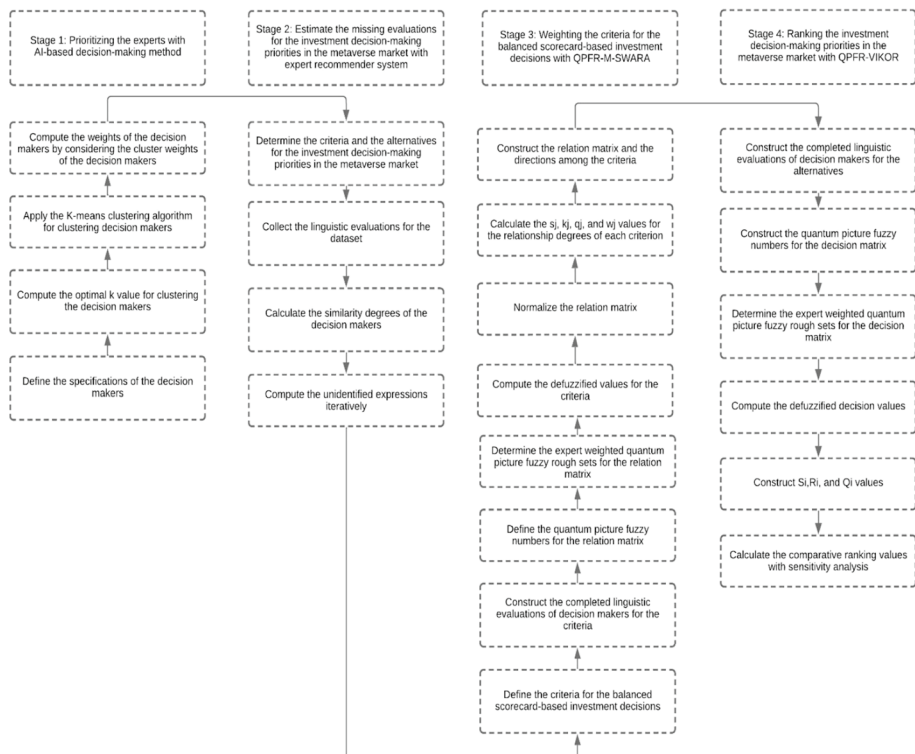


Fig. 1 Flowchart

3.1 QPFR sets with golden cuts

Fuzzy set theory is the branch of mathematics proposed to address uncertainty (Eti and Yüksel, 2024). Fuzzy sets are used to consider the uncertainty in the multi-criteria decision-making analysis process (Chusi et al., 2024). QPFR sets with golden cuts used in the study is a version of fuzzy logic obtained by integrating quantum mechanics, golden ratio, and rough sets. In this way, subjective and uncertain evaluations for the opinions of different types of experts can be decreased. Details of the process of obtaining QPFR sets are discussed in this section.

In quantum mechanics studies, whether a particle with mass is in certain coordinates or has momentum is determined by wave function. In other words, the square of the wave function equals its value relative to the particle. The wave function of a particle with mass is represented by a complex function with amplitude and phase angle. In this function, the amplitude refers to magnitude of the wave function, while phase angle refers to the phase shift of the wave function (Du et al. 2024; Hou 2024; Kou et al. 2023a, b). Details about the function are given in Eqs. (1)–(3) (Fan et al. 2023; Carayannis et al. 2023).

$$Q(|u\rangle) = \varphi e^{i\theta} \quad (1)$$

$$|C\rangle = \{|u_1\rangle, |u_2\rangle, \dots, |u_n\rangle\} \quad (2)$$

$$\sum_{|u\rangle \in |C\rangle} |Q(|u\rangle)| = 1 \quad (3)$$

In the realm of quantum mechanics, C represents a collection of exhaustive events represented by $|u_i\rangle$. The squared magnitude of the wave function, $|Q(|u\rangle)| = \varphi^2$, provides the amplitude-based result for the probability of occurrence of event $|u\rangle$ as described by the principles of quantum logic. The value of φ^2 must lie within the range of 0 to 1, and θ^2 represents the phase angle of event $|u\rangle$. Furthermore, $|\varphi_1|^2$ serves as the degree of belief in event $|u\rangle$, with θ representing its phase angle, which can range from 0 to 360° .

Picture fuzzy (PFS) sets in QPFR sets are one type of intuitionistic fuzzy logic. PFS enables comprehensive fuzzy-based opinions in decision-making analyses (Ahmad et al. 2024; Rani et al. 2024). For this, PFSs include refusal, negative, neutral, and positive membership values of fuzzy sets in a space. The conventional fuzzy sets are defined by Eq. (4).

$$A = \{\langle x, \mu_A(x) \rangle | x \in X\} \quad (4)$$

where A defines the fuzzy sets, X is a universe of discourse and μ_A is the membership degree of x in the fuzzy set A . $\mu_A : X \rightarrow [0,1]$.

Intuitionistic fuzzy sets are defined by adding the non-membership function to conventional fuzzy sets, as shown in Eq. (5).

$$A = \{\langle x, \mu_A(x), \nu_A(x) \rangle | x \in X\} \quad (5)$$

where ν_A represents the non-membership function of A , $0 \leq \mu_A(x) + \nu_A(x) \leq 1, \forall x \in X$. PFSs are shown in Eq. (6) and are the version of intuitionistic fuzzy sets with neutral (n_A) and refusal (h_A) functions added (Kahraman 2024).

$$A = \{\langle x, \mu_A(x), n_A(x), \nu_A(x), h_A(x) \rangle | x \in X\} \quad (6)$$

here, $\mu_A(x) + n_A(x) + v_A(x) + h_A(x) = 1, \forall x \in X$. With PFS, experts can make positive and negative evaluations, as well as evaluations with a degree of neutral and refusal. Thus, the use of PFS is adept at producing more consistent results than previous fuzzy sets in real-world decision-making analysis. General operations of PFS are presented in Eqs. (7)–(11).

$$A \subseteq B \text{ if } \mu_A(x) \leq \mu_B(x) \text{ and } n_A(x) \leq n_B(x) \text{ and } v_A(x) \geq v_B(x), \forall x \in X \quad (7)$$

$$A = B \text{ if } A \subseteq B \text{ and } B \subseteq A \quad (8)$$

$$A \cup B = \{ (x, \max(\mu_A(x), \mu_B(x)), \min(n_A(x), n_B(x)), \min(v_A(x), v_B(x))) | x \in X \} \quad (9)$$

$$A \cap B = \{ (x, \min(\mu_A(x), \mu_B(x)), \min(n_A(x), n_B(x)), \max(v_A(x), v_B(x))) | x \in X \} \quad (10)$$

$$coA = \bar{A} = \{ (x, v_A(x), n_A(x), \mu_A(x)) | x \in X \} \quad (11)$$

Rough numbers are part of rough set theory, which is preferred to minimize subjective and uncertain evaluations in decision analysis. Rough numbers include a rough boundary interval ($Bnd(C_i)$), in addition to lower ($\underline{Apr}(C_i)$) and upper ($\overline{Apr}(C_i)$) limits (Bozanic et al. 2023). The components of C_i are calculated with the help of Eqs. (12)–(14).

$$\underline{Apr}(C_i) = \cup \left\{ Y \in \frac{X}{R(Y)} \leq C_i \right\} \quad (12)$$

$$\overline{Apr}(C_i) = \cup \left\{ Y \in \frac{X}{R(Y)} \geq C_i \right\} \quad (13)$$

$$Bnd(C_i) = \cup \left\{ Y \in \frac{X}{R(Y)} \neq C_i \right\} \quad (14)$$

However, lower ($\underline{Lim}(C_i)$), upper ($\overline{Lim}(C_i)$) limits and the rough number ($RN(C_i)$) of C_i are shown in Eq. (15)–(17).

$$\underline{Lim}(C_i) = \sqrt[N_L]{\prod_{i=1}^{N_L} Y \in \underline{Apr}(C_i)} \quad (15)$$

$$\overline{Lim}(C_i) = \sqrt[N_U]{\prod_{i=1}^{N_U} Y \in \overline{Apr}(C_i)} \quad (16)$$

$$RN(C_i) = [\underline{Lim}(C_i), \overline{Lim}(C_i)] \quad (17)$$

Here N_L and N_U of $\underline{Apr}(C_i)$ and $\overline{Apr}(C_i)$ are amount of elements. In the article, it is aimed to combine quantum mechanics, picture fuzzy sets and rough numbers introduced above. In this way, more comprehensive and realistic results can be obtained, as well as minimizing subjective and uncertainty. Quantum picture fuzzy rough sets (QPFRSs) are depicted with Eq. (18).

$$|C_A > = \left\{ \begin{array}{l} \langle u, (\underline{\text{Lim}}(C_{i\mu_A}), \overline{\text{Lim}}(C_{i\mu_A})) \rangle(u), \langle \underline{\text{Lim}}(C_{in_A}), \overline{\text{Lim}}(C_{in_A}) \rangle(u), \\ \langle \underline{\text{Lim}}(C_{iv_A}), \overline{\text{Lim}}(C_{iv_A}) \rangle(u), \langle \underline{\text{Lim}}(C_{ih_A}), \overline{\text{Lim}}(C_{ih_A}) \rangle(u) \mid u \in 2^{|C_A|} \end{array} \right\} \quad (18)$$

where, $C_{i\mu_A}$ is the membership, C_{in_A} means information about the neutral, C_{iv_A} represents the non-membership, and C_{ih_A} is about the refusal. The components of PFRN are computed by Eqs. (19)–(34).

$$\underline{\text{Lim}}(C_{i\mu_A}) = \frac{1}{N_{L\mu_A}} \sum_{i=1}^{N_{L\mu_A}} Y \in \underline{\text{Apr}}(C_{i\mu_A}) \quad (19)$$

$$\underline{\text{Lim}}(C_{in_A}) = \frac{1}{N_{Ln_A}} \sum_{i=1}^{N_{Ln_A}} Y \in \underline{\text{Apr}}(C_{in_A}) \quad (20)$$

$$\underline{\text{Lim}}(C_{iv_A}) = \frac{1}{N_{Lv_A}} \sum_{i=1}^{N_{Lv_A}} Y \in \underline{\text{Apr}}(C_{iv_A}) \quad (21)$$

$$\underline{\text{Lim}}(C_{ih_A}) = \frac{1}{N_{L\pi_A}} \sum_{i=1}^{N_{L\pi_A}} Y \in \underline{\text{Apr}}(C_{ih_A}) \quad (22)$$

$$\overline{\text{Lim}}(C_{i\mu_A}) = \frac{1}{N_{U\mu_A}} \sum_{i=1}^{N_{U\mu_A}} Y \in \overline{\text{Apr}}(C_{i\mu_A}) \quad (23)$$

$$\overline{\text{Lim}}(C_{in_A}) = \frac{1}{N_{Un_A}} \sum_{i=1}^{N_{Un_A}} Y \in \overline{\text{Apr}}(C_{in_A}) \quad (24)$$

$$\overline{\text{Lim}}(C_{iv_A}) = \frac{1}{N_{Uv_A}} \sum_{i=1}^{N_{Uv_A}} Y \in \overline{\text{Apr}}(C_{iv_A}) \quad (25)$$

$$\overline{\text{Lim}}(C_{ih_A}) = \frac{1}{N_{U\pi_A}} \sum_{i=1}^{N_{U\pi_A}} Y \in \overline{\text{Apr}}(C_{ih_A}) \quad (26)$$

$$\underline{\text{Apr}}(C_{i\mu_A}) = \cup \left\{ Y \in \frac{X}{\widetilde{R}(Y)} \leq C_{i\mu_A} \right\} \quad (27)$$

$$\underline{\text{Apr}}(C_{in_A}) = \cup \left\{ Y \in \frac{X}{\widetilde{R}(Y)} \leq C_{in_A} \right\} \quad (28)$$

$$\underline{Apr}(C_{iv_A}) = \cup \left\{ Y \in \frac{X}{\tilde{R}(Y)} \leq C_{iv_A} \right\} \quad (29)$$

$$\underline{Apr}(C_{ih_A}) = \cup \left\{ Y \in \frac{X}{\tilde{R}(Y)} \leq C_{ih_A} \right\} \quad (30)$$

$$\overline{Apr}(C_{i\mu_A}) = \cup \left\{ Y \in \frac{X}{\tilde{R}(Y)} \leq C_{i\mu_A} \right\} \quad (31)$$

$$\overline{Apr}(C_{in_A}) = \cup \left\{ Y \in \frac{X}{\tilde{R}(Y)} \leq C_{in_A} \right\} \quad (32)$$

$$\overline{Apr}(C_{iv_A}) = \cup \left\{ Y \in \frac{X}{\tilde{R}(Y)} \leq C_{iv_A} \right\} \quad (33)$$

$$\overline{Apr}(C_{ih_A}) = \cup \left\{ Y \in \frac{X}{\tilde{R}(Y)} \leq C_{ih_A} \right\} \quad (34)$$

where $N_{L\mu_A}, N_{Ln_A}, N_{Lv_A}, N_{Lh_A}$ are the number of objects in $\underline{Apr}(C_{i\mu_A}), \underline{Apr}(C_{in_A}), \underline{Apr}(C_{iv_A}), \underline{Apr}(C_{ih_A})$ respectively while $N_{U\mu_A}, N_{Un_A}, N_{Uv_A}, N_{Uh_A}$ are determined for $\overline{Apr}(C_{i\mu_A}), \overline{Apr}(C_{in_A}), \overline{Apr}(C_{iv_A}), \overline{Apr}(C_{ih_A})$. $\tilde{C}_i = (C_{i\mu_A}, C_{in_A}, C_{iv_A}, C_{ih_A})$ and $\tilde{R}$ is the collection of $\{\tilde{C}_1, \tilde{C}_2, \dots, \tilde{C}_n\}$. $C_{i\mu_A}, C_{in_A}, C_{iv_A}, C_{ih_A}$ are the PFSs of class $\tilde{C}_i$. In the next definition, it is represented by the amplitude and angle functions of QPFS by Eqs. (35) and (36).

$$C = [C_\mu \cdot e^{j2\pi \cdot \alpha}, C_n \cdot e^{j2\pi \cdot \gamma}, C_v \cdot e^{j2\pi \cdot \beta}, C_h \cdot e^{j2\pi \cdot T}] \quad (35)$$

$$\varphi^2 = |C_\mu(|u_i\rangle)| \quad (36)$$

The components of quantum functions, denoted with C_μ, C_n, C_v and C_h correspond to phase angles α, γ, β, T . The components of the membership function C_μ of QFS is determined with φ^2 . In multi-objective optimization problems, the golden ratio is used to maximize the benefit function while minimizing the cost function. The golden ratio (G) is a mathematical constant approximately equal to 1.618 and is denoted ϕ (Phi). Due to this feature of the golden ratio, it can be used to ideally define membership and non-membership values in fuzzy sets. The expression of the two components with the G through amplitude is given in Eqs. (37) and (38).

$$C_n = \frac{C_\mu}{G} \quad (37)$$

$$C_h = \frac{C_v}{G} \quad (38)$$

Finally, the phase angle of the proposed functions is displayed. The phase angle of the membership degrees for the probability of event $|u_i\rangle$ in the realm of QPFS is denoted using Eqs. (39)–(41).

$$\alpha = |C_\mu(|u_i\rangle)| \quad (39)$$

$$\gamma = \frac{\alpha}{G} \quad (40)$$

$$T = \frac{\beta}{G} \quad (41)$$

X_1 and X_2 are two universes, and $\tilde{A}_c$ and $\tilde{B}_c$, respectively represented by

$$\left(\begin{aligned} & \left[\underline{\text{Lim}}(C_{\mu_{\tilde{A}}}), \overline{\text{Lim}}(C_{\mu_{\tilde{A}}}) \right] e^{j2\pi \cdot \left[\left(\frac{\alpha_{\tilde{A}}}{2\pi} \right), \left(\frac{\tilde{\alpha}_{\tilde{A}}}{2\pi} \right) \right]}, \left[\underline{\text{Lim}}(C_{n_{\tilde{A}}}), \overline{\text{Lim}}(C_{n_{\tilde{A}}}) \right] e^{j2\pi \cdot \left[\left(\frac{\gamma_{\tilde{A}}}{2\pi} \right), \left(\frac{\tilde{\gamma}_{\tilde{A}}}{2\pi} \right) \right]}, \\ & \left[\underline{\text{Lim}}(C_{v_{\tilde{A}}}), \overline{\text{Lim}}(C_{v_{\tilde{A}}}) \right] e^{j2\pi \cdot \left[\left(\frac{\beta_{\tilde{A}}}{2\pi} \right), \left(\frac{\tilde{\beta}_{\tilde{A}}}{2\pi} \right) \right]}, \left[\underline{\text{Lim}}(C_{h_{\tilde{A}}}), \overline{\text{Lim}}(C_{h_{\tilde{A}}}) \right] e^{j2\pi \cdot \left[\left(\frac{\tau_{\tilde{A}}}{2\pi} \right), \left(\frac{\tilde{\tau}_{\tilde{A}}}{2\pi} \right) \right]} \end{aligned} \right),$$

$$\left(\begin{aligned} & \left[\underline{\text{Lim}}(C_{\mu_{\tilde{B}}}), \overline{\text{Lim}}(C_{\mu_{\tilde{B}}}) \right] e^{j2\pi \cdot \left[\left(\frac{\alpha_{\tilde{B}}}{2\pi} \right), \left(\frac{\tilde{\alpha}_{\tilde{B}}}{2\pi} \right) \right]}, \left[\underline{\text{Lim}}(C_{n_{\tilde{B}}}), \overline{\text{Lim}}(C_{n_{\tilde{B}}}) \right] e^{j2\pi \cdot \left[\left(\frac{\gamma_{\tilde{B}}}{2\pi} \right), \left(\frac{\tilde{\gamma}_{\tilde{B}}}{2\pi} \right) \right]}, \\ & \left[\underline{\text{Lim}}(C_{v_{\tilde{B}}}), \overline{\text{Lim}}(C_{v_{\tilde{B}}}) \right] e^{j2\pi \cdot \left[\left(\frac{\beta_{\tilde{B}}}{2\pi} \right), \left(\frac{\tilde{\beta}_{\tilde{B}}}{2\pi} \right) \right]}, \left[\underline{\text{Lim}}(C_{h_{\tilde{B}}}), \overline{\text{Lim}}(C_{h_{\tilde{B}}}) \right] e^{j2\pi \cdot \left[\left(\frac{\tau_{\tilde{B}}}{2\pi} \right), \left(\frac{\tilde{\tau}_{\tilde{B}}}{2\pi} \right) \right]} \end{aligned} \right)$$

and, Let X_1 and X_2 be two different QPFRS. The operations of QPFRN are formulated in the following set of Eqs. (42)–(45). For $\lambda > 0$,

$$\lambda * \tilde{A}_c = \left\{ \begin{aligned} & \left[\underline{\text{Lim}}(C_{\mu_{\tilde{A}}})^\lambda, \overline{\text{Lim}}(C_{\mu_{\tilde{A}}})^\lambda \right] e^{j2\pi \cdot \left[\left(\frac{\alpha_{\tilde{A}}}{2\pi} \right)^\lambda, \left(\frac{\tilde{\alpha}_{\tilde{A}}}{2\pi} \right)^\lambda \right]}, \left[\underline{\text{Lim}}(C_{n_{\tilde{A}}})^\lambda, \overline{\text{Lim}}(C_{n_{\tilde{A}}})^\lambda \right] e^{j2\pi \cdot \left[\left(\frac{\gamma_{\tilde{A}}}{2\pi} \right)^\lambda, \left(\frac{\tilde{\gamma}_{\tilde{A}}}{2\pi} \right)^\lambda \right]}, \\ & \left[\underline{\text{Lim}}(C_{v_{\tilde{A}}})^\lambda, \overline{\text{Lim}}(C_{v_{\tilde{A}}})^\lambda \right] e^{j2\pi \cdot \left[\left(\frac{\beta_{\tilde{A}}}{2\pi} \right)^\lambda, \left(\frac{\tilde{\beta}_{\tilde{A}}}{2\pi} \right)^\lambda \right]}, \left[\underline{\text{Lim}}(C_{h_{\tilde{A}}})^\lambda, \overline{\text{Lim}}(C_{h_{\tilde{A}}})^\lambda \right] e^{j2\pi \cdot \left[\left(\frac{\tau_{\tilde{A}}}{2\pi} \right)^\lambda, \left(\frac{\tilde{\tau}_{\tilde{A}}}{2\pi} \right)^\lambda \right]} \end{aligned} \right\} \quad (42)$$

$$\tilde{A}_c^\lambda = \left\{ \begin{aligned} & \left[\underline{\text{Lim}}(C_{\mu_{\tilde{A}}})^\lambda, \overline{\text{Lim}}(C_{\mu_{\tilde{A}}})^\lambda \right] e^{j2\pi \cdot \left[\left(\frac{\alpha_{\tilde{A}}}{2\pi} \right)^\lambda, \left(\frac{\tilde{\alpha}_{\tilde{A}}}{2\pi} \right)^\lambda \right]}, \left[\underline{\text{Lim}}(C_{n_{\tilde{A}}})^\lambda, \overline{\text{Lim}}(C_{n_{\tilde{A}}})^\lambda \right] e^{j2\pi \cdot \left[\left(\frac{\gamma_{\tilde{A}}}{2\pi} \right)^\lambda, \left(\frac{\tilde{\gamma}_{\tilde{A}}}{2\pi} \right)^\lambda \right]}, \\ & \left[\underline{\text{Lim}}(C_{v_{\tilde{A}}})^\lambda, \overline{\text{Lim}}(C_{v_{\tilde{A}}})^\lambda \right] e^{j2\pi \cdot \left[\left(\frac{\beta_{\tilde{A}}}{2\pi} \right)^\lambda, \left(\frac{\tilde{\beta}_{\tilde{A}}}{2\pi} \right)^\lambda \right]}, \left[\underline{\text{Lim}}(C_{h_{\tilde{A}}})^\lambda, \overline{\text{Lim}}(C_{h_{\tilde{A}}})^\lambda \right] e^{j2\pi \cdot \left[\left(\frac{\tau_{\tilde{A}}}{2\pi} \right)^\lambda, \left(\frac{\tilde{\tau}_{\tilde{A}}}{2\pi} \right)^\lambda \right]} \end{aligned} \right\} \quad (43)$$

$$\tilde{A}_c \cup \tilde{B}_c = \left\{ \begin{aligned} & \left[\min \left(\underline{\text{Lim}}(C_{\mu_{\tilde{A}}}) e^{j2\pi \cdot \left(\frac{\alpha_{\tilde{A}}}{2\pi} \right)}, \underline{\text{Lim}}(C_{\mu_{\tilde{B}}}) e^{j2\pi \cdot \left(\frac{\alpha_{\tilde{B}}}{2\pi} \right)} \right), \max \left(\overline{\text{Lim}}(C_{\mu_{\tilde{A}}}) e^{j2\pi \cdot \left(\frac{\tilde{\alpha}_{\tilde{A}}}{2\pi} \right)}, \overline{\text{Lim}}(C_{\mu_{\tilde{B}}}) e^{j2\pi \cdot \left(\frac{\tilde{\alpha}_{\tilde{B}}}{2\pi} \right)} \right) \right], \\ & \left[\min \left(\underline{\text{Lim}}(C_{n_{\tilde{A}}}) e^{j2\pi \cdot \left(\frac{\gamma_{\tilde{A}}}{2\pi} \right)}, \underline{\text{Lim}}(C_{n_{\tilde{B}}}) e^{j2\pi \cdot \left(\frac{\gamma_{\tilde{B}}}{2\pi} \right)} \right), \max \left(\overline{\text{Lim}}(C_{n_{\tilde{A}}}) e^{j2\pi \cdot \left(\frac{\tilde{\gamma}_{\tilde{A}}}{2\pi} \right)}, \overline{\text{Lim}}(C_{n_{\tilde{B}}}) e^{j2\pi \cdot \left(\frac{\tilde{\gamma}_{\tilde{B}}}{2\pi} \right)} \right) \right], \\ & \left[\min \left(\underline{\text{Lim}}(C_{v_{\tilde{A}}}) e^{j2\pi \cdot \left(\frac{\beta_{\tilde{A}}}{2\pi} \right)}, \underline{\text{Lim}}(C_{v_{\tilde{B}}}) e^{j2\pi \cdot \left(\frac{\beta_{\tilde{B}}}{2\pi} \right)} \right), \max \left(\overline{\text{Lim}}(C_{v_{\tilde{A}}}) e^{j2\pi \cdot \left(\frac{\tilde{\beta}_{\tilde{A}}}{2\pi} \right)}, \overline{\text{Lim}}(C_{v_{\tilde{B}}}) e^{j2\pi \cdot \left(\frac{\tilde{\beta}_{\tilde{B}}}{2\pi} \right)} \right) \right], \\ & \left[\min \left(\underline{\text{Lim}}(C_{h_{\tilde{A}}}) e^{j2\pi \cdot \left(\frac{\tau_{\tilde{A}}}{2\pi} \right)}, \underline{\text{Lim}}(C_{h_{\tilde{B}}}) e^{j2\pi \cdot \left(\frac{\tau_{\tilde{B}}}{2\pi} \right)} \right), \max \left(\overline{\text{Lim}}(C_{h_{\tilde{A}}}) e^{j2\pi \cdot \left(\frac{\tilde{\tau}_{\tilde{A}}}{2\pi} \right)}, \overline{\text{Lim}}(C_{h_{\tilde{B}}}) e^{j2\pi \cdot \left(\frac{\tilde{\tau}_{\tilde{B}}}{2\pi} \right)} \right) \right] \end{aligned} \right\} \quad (44)$$

$$\tilde{A}_c \cap \tilde{B}_c = \left[\begin{array}{l} \left[\max \left(\underline{\text{Lim}}(C_{\mu_A^-}) e^{j2\pi \left(\frac{\alpha_A^-}{2\pi} \right)}, \underline{\text{Lim}}(C_{\mu_B^-}) e^{j2\pi \left(\frac{\alpha_B^-}{2\pi} \right)}, \min \left(\overline{\text{Lim}}(C_{\mu_A^-}) e^{j2\pi \left(\frac{\alpha_A^-}{2\pi} \right)}, \overline{\text{Lim}}(C_{\mu_B^-}) e^{j2\pi \left(\frac{\alpha_B^-}{2\pi} \right)} \right) \right], \\ \left[\max \left(\underline{\text{Lim}}(C_{\mu_A^+}) e^{j2\pi \left(\frac{\gamma_A^+}{2\pi} \right)}, \underline{\text{Lim}}(C_{\mu_B^+}) e^{j2\pi \left(\frac{\gamma_B^+}{2\pi} \right)}, \min \left(\overline{\text{Lim}}(C_{\mu_A^+}) e^{j2\pi \left(\frac{\gamma_A^+}{2\pi} \right)}, \overline{\text{Lim}}(C_{\mu_B^+}) e^{j2\pi \left(\frac{\gamma_B^+}{2\pi} \right)} \right) \right], \\ \left[\max \left(\underline{\text{Lim}}(C_{\nu_A^-}) e^{j2\pi \left(\frac{\beta_A^-}{2\pi} \right)}, \underline{\text{Lim}}(C_{\nu_B^-}) e^{j2\pi \left(\frac{\beta_B^-}{2\pi} \right)}, \min \left(\overline{\text{Lim}}(C_{\nu_A^-}) e^{j2\pi \left(\frac{\beta_A^-}{2\pi} \right)}, \overline{\text{Lim}}(C_{\nu_B^-}) e^{j2\pi \left(\frac{\beta_B^-}{2\pi} \right)} \right) \right], \\ \left[\max \left(\underline{\text{Lim}}(C_{h_A^-}) e^{j2\pi \left(\frac{\tau_A^-}{2\pi} \right)}, \underline{\text{Lim}}(C_{h_B^-}) e^{j2\pi \left(\frac{\tau_B^-}{2\pi} \right)}, \min \left(\overline{\text{Lim}}(C_{h_A^-}) e^{j2\pi \left(\frac{\tau_A^-}{2\pi} \right)}, \overline{\text{Lim}}(C_{h_B^-}) e^{j2\pi \left(\frac{\tau_B^-}{2\pi} \right)} \right) \right] \end{array} \right] \quad (45)$$

3.2 AI-based decision-making for experts prioritization

In the literature, identifying experts and assigning importance weights for expert evaluation-based analyzes are subject to criticism. Artificial intelligence is used to prioritize experts in the study. Using the k-means clustering algorithm, expert prioritization is analyzed, while the elbow method is used to determine the ideal number of clusters. The steps of the method are exhibited in the following formulations.

In the *Step 1*, specifications of the experts are defined. Elbow method is used for defined of the number of optimal clusters. Drawing the Within-Cluster Sum of Squares (WCSS) with clusters' number (k) indicates that this method identifies the elbow-shaped point on the graph that represents the point at which adding more clusters no longer significantly increases the efficacy of the model. As a result, the elbow technique acts as a strategic navigator that helps us balance the level of clustering by diminishing benefits of additional complexity. This guarantees that clusters' number chosen captures the information collected in the population, offering a strong basis for the other phases in our prioritization process. In *Step 2*, WCSS is calculated for the various values of k with Eq. (46).

$$WCSS = \sum_{j=1}^k \sum_{x_i \in C_j} d(x_i, c_j)^2 \quad (46)$$

where k represents the amount of clusters, C_j means the set of elements in cluster j , x_i is a element, c_j is the cluster center of cluster j , and $d(x_i, c_j)$ equals the Euclidean distance (Costantiello and Leogrande 2024). In the graphical representation where the WCSS (Within-Cluster Sum of Squares) values and the corresponding values of k are displayed, the elbow point is identified as the optimal k value, signifying the point at which the reduction in WCSS decelerates. During *Step 3* of the prioritization process for AI-based decision makers, the K-means algorithm is employed to cluster experts. The optimal value of k is utilized to establish the initial cluster centers, denoted $asc_1, c_2, \dots, c_k$. Subsequently, each element x_i is assigned to the nearest cluster center, defining the cluster assignments based on Eq. (47).

$$d(x_i, x_j) = \sqrt{\sum_{l=1}^n (x_{il} - x_{jl})^2} \quad (47)$$

where n is amount of components of dataset. The cluster of each x_i is stated with a_i , here $a_i = j$ is that x_i relatives to j -cluster (Kong et al. 2024; Yang 2024). Center of clusters is revised by taking the mean of elements in each cluster by Eq. (48).

$$c_j = \frac{1}{|C_j|} \sum_{x_i \in C_j} x_i \quad (48)$$

where C_j is the set of elements in cluster j , and $|C_j|$ is the number of elements in cluster j . Equations (47) and (48) are calculated until a maximum number of iterations is reached (Liu et al. 2024). In *Step 4*, the experts' weights are computed with considering the weights of clusters. The mean standard deviations for each cluster are determined through the application of Eqs. (49) to (51).

$$s_j = \frac{1}{n} \sum_{l=1}^n \sigma_{jl} \quad (49)$$

$$\sigma_{jl} = \sqrt{\frac{1}{|C_j|} \sum_{x_i \in C_j} (x_{il} - \bar{x}_{jl})^2} \quad (50)$$

$$\bar{x}_{jl} = \frac{1}{|C_j|} \sum_{x_i \in C_j} x_{il} \quad (51)$$

where, s_j is the mean standard deviation of cluster j . n is the number of features or dimensions of the data, and σ_{jl} is the standard deviation of feature l in cluster j . $\bar{x}_{jl}$ is the mean of feature l in cluster j .

The cluster weights (w_j) are found by Eq. (52).

$$w_j = |C_j| \times s_j \quad (52)$$

where $|C_j|$ is the size of cluster j . The experts' weight is computed by Eq. (53).

$$w_{tj} = \frac{1}{|C_j|} \frac{w_j}{\sum_{w_j \in C_j} w_j} \quad (53)$$

where, t explains the number of experts w_{tj} refers the weight of t -expert in j -cluster.

3.3 Recommendation system with collaborative filtering

Collaborative filtering technique is used to complete missing values in a data set. In this way, it is possible for the results to be close to reality. Thanks to this method, incomplete answers can be completed if the experts' questions are not answered completely. In this way, experts do not give answers that they are not sure about, thus preventing the

quality of analysis results from being negatively affected. The steps of the method are given below.

In *Step 5*, alternatives and criteria for investment decision-making priorities in the metaverse market are determined. *Step 6* is about collecting linguistic expressions for the dataset. At the end of *Step 7*, the similarity degrees of the experts are calculated using Eqs. (54) and (55) where $r_{u,i}/r_{v,i}$ explain the rating degrees of experts and $\bar{r}_u$ and $\bar{r}_v$ refer to the averaged values.

$$\text{sim}(u, v) = \frac{\sum_{i \in I} (r_{u,i} - \bar{r}_u)(r_{v,i} - \bar{r}_v)}{\sqrt{\sum_{i \in I} (r_{u,i} - \bar{r}_u)^2} \sqrt{\sum_{i \in I} (r_{v,i} - \bar{r}_v)^2}} \quad (54)$$

$$p_{u,i} = \frac{\sum_{j \in S} \text{sim}(u, v) r_{u,j}}{\sum_{j \in S} |\text{sim}(u, v)|} \quad (55)$$

In *Step 8*, the unidentified expressions iteratively are computed.

3.4 M-SWARA using QPFRS

SWARA is preferred for weighting criteria in MCDM. M-SWARA is a version of the SWARA. Proposed model is detailed below.

Step 9 covers defining balanced scorecard-based investment decisions. Criteria affecting metaverse investment are defined from literature. In *Step 10*, the completed linguistic evaluations of decision makers for the criteria are constructed. *Step 11* involves defining QPFMs for the relationship matrix. QPFR relation matrix is created by considering the linguistic evaluations of experts and the quantum spherical fuzzy numbers. $C_{\mu_A} C = [C_{ij}]_{n \times n}$ are the relationship each criterion. C_{ij} is given in Eq. (56) (Mikhaylov et al. 2024).

$$C_k = \begin{bmatrix} 0 & C_{12} & \cdots & \cdots & C_{1n} \\ C_{21} & 0 & \cdots & \cdots & C_{2n} \\ \vdots & \vdots & \ddots & \cdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ C_{n1} & C_{n2} & \cdots & \cdots & 0 \end{bmatrix} \quad (56)$$

where C determines QPFR direct relation matrix.

$$C_{ij} = \left[\begin{array}{l} \left[\underline{\text{Lim}}(C_{\mu_{ij}}), \overline{\text{Lim}}(C_{\mu_{ij}}) \right] e^{j2\pi \cdot \left[\left(\frac{\alpha_{ij}}{2\pi} \right), \left(\frac{\bar{\alpha}_{ij}}{2\pi} \right) \right]}, \left[\underline{\text{Lim}}(C_{n_{ij}}), \overline{\text{Lim}}(C_{n_{ij}}) \right] e^{j2\pi \cdot \left[\left(\frac{\gamma_{ij}}{2\pi} \right), \left(\frac{\bar{\gamma}_{ij}}{2\pi} \right) \right]}, \\ \left[\underline{\text{Lim}}(C_{v_{ij}}), \overline{\text{Lim}}(C_{v_{ij}}) \right] e^{j2\pi \cdot \left[\left(\frac{\beta_{ij}}{2\pi} \right), \left(\frac{\bar{\beta}_{ij}}{2\pi} \right) \right]}, \left[\underline{\text{Lim}}(C_{h_{ij}}), \overline{\text{Lim}}(C_{h_{ij}}) \right] e^{j2\pi \cdot \left[\left(\frac{\tau_{ij}}{2\pi} \right), \left(\frac{\bar{\tau}_{ij}}{2\pi} \right) \right]} \end{array} \right], \text{ and}$$

k is the number of decision makers. In *Step 12*, the expert weighted QPFRS for relation matrix is determined. Expert weighted QPFRN are calculated in Eq. (57).

$$w_k \times C \quad (57)$$

where w_k is the weights of experts. QPFRS is defined. The C of experts is computed with QPFRN in Eq. (58).

$$C = \begin{pmatrix} \left[\min_{i=1}^k \left(\underline{\text{Lim}}(C_{\mu_{ij}}) \right), \max_{i=1}^k \left(\overline{\text{Lim}}(C_{\mu_{ij}}) \right) \right] e^{j2\pi \cdot \left[\min_{i=1}^k \left(\frac{\alpha}{2\pi} \right), \max_{i=1}^k \left(\frac{\alpha_{ij}}{2\pi} \right) \right]}, \\ \left[\min_{i=1}^k \left(\underline{\text{Lim}}(C_{n_{ij}}) \right), \max_{i=1}^k \left(\overline{\text{Lim}}(C_{n_{ij}}) \right) \right] e^{j2\pi \cdot \left[\min_{i=1}^k \left(\frac{\gamma}{2\pi} \right), \max_{i=1}^k \left(\frac{\gamma_{ij}}{2\pi} \right) \right]}, \\ \left[\min_{i=1}^k \left(\underline{\text{Lim}}(C_{v_{ij}}) \right), \max_{i=1}^k \left(\overline{\text{Lim}}(C_{v_{ij}}) \right) \right] e^{j2\pi \cdot \left[\min_{i=1}^k \left(\frac{\beta}{2\pi} \right), \max_{i=1}^k \left(\frac{\beta_{ij}}{2\pi} \right) \right]}, \\ \left[\min_{i=1}^k \left(\underline{\text{Lim}}(C_{h_{ij}}) \right), \max_{i=1}^k \left(\overline{\text{Lim}}(C_{h_{ij}}) \right) \right] e^{j2\pi \cdot \left[\min_{i=1}^k \left(\frac{\tau}{2\pi} \right), \max_{i=1}^k \left(\frac{\tau_{ij}}{2\pi} \right) \right]} \end{pmatrix} \quad (58)$$

In *Step 13*, defuzzified values are computed. *Defc* of QPFRSs is computed by Eq. (59).

$$\text{Defc}_i = \frac{\left(\frac{\underline{\text{Lim}}(C_{\mu_i}) - \underline{\text{Lim}}(C_{n_i}) + \underline{\text{Lim}}(C_{\mu_i}) \cdot (\underline{\text{Lim}}(C_{v_i}) - \underline{\text{Lim}}(C_{h_i})) + \left(\frac{\alpha}{2\pi} \right) - \left(\frac{\gamma}{2\pi} \right) + \left(\frac{\alpha}{2\pi} \right) \cdot \left(\left(\frac{\beta}{2\pi} \right) - \left(\frac{\tau}{2\pi} \right) \right) + \right.}{2} \left. \frac{\overline{\text{Lim}}(C_{\mu_i}) - \overline{\text{Lim}}(C_{n_i}) + \overline{\text{Lim}}(C_{\mu_i}) \cdot (\overline{\text{Lim}}(C_{v_i}) - \overline{\text{Lim}}(C_{h_i})) + \left(\frac{\alpha_{ij}}{2\pi} \right) - \left(\frac{\gamma_{ij}}{2\pi} \right) + \left(\frac{\alpha_{ij}}{2\pi} \right) \cdot \left(\left(\frac{\beta_{ij}}{2\pi} \right) - \left(\frac{\tau_{ij}}{2\pi} \right) \right) \right) \quad (59)$$

Step 14 is process of founding the normalized relationship matrix. With *Step 15*, s_j , k_j , q_j , and w_j are computed using Eqs. (60)–(62).

$$k_j = \begin{cases} 1 & j = 1 \\ s_j + 1 & j > 1 \end{cases} \quad (60)$$

$$q_j = \begin{cases} 1 & j = 1 \\ \frac{q_{j-1}}{k_j} & j > 1 \end{cases} \quad (61)$$

If $s_{j-1} = s_j$, $q_{j-1} = q_j$; If $s_j = 0$, $k_{j-1} = k_j$

$$w_j = \frac{q_j}{\sum_{k=1}^n q_k} \quad (62)$$

s_j is the comparative importance rate with quantum picture fuzzy rough numbers and provides the importance value of the criterion c_j on the following criterion c_{j+1} . k_j is the coefficient value of the s_j and q_j defines the recalculated weight of k_j . w_j gives information about the weights of the criteria under the fuzzy sets. The significance degrees of the criteria are sorted in descending order.

Step 16 involves the generation of the relation matrix and the determination of the directional relationships between the criteria. Stable matrix is obtained by the power of $2t+1$. t takes very largest. The results of w_j are established with the process. The degrees of impact-relation for the criteria are established by applying a cutoff value. This is the mean of relation matrix (Mikhaylov et al. 2023).

3.5 VIKOR with QPFRS

VIKOR is a consensus-based approach to decision-making that is centered on getting near to the best answer. By taking the criteria into account, the VIKOR approach attempts to rank the alternatives. In the VIKOR technique, a compromise solution entails agreement with reciprocal concessions (Mishra et al. 2023).

In *Step 17*, the completed linguistic evaluations of experts for the alternatives are constructed. *Step 18* involves creating the QPFNs of the decision matrix (X). In *Step 19*, QPFRSs are defined for X. The X is given in Eq. (63).

$$X_k = \begin{bmatrix} 0 & X_{12} & \cdots & \cdots & X_{1m} \\ X_{21} & 0 & \cdots & \cdots & X_{2m} \\ \vdots & \vdots & \ddots & \cdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ X_{n1} & X_{n2} & \cdots & \cdots & 0 \end{bmatrix} \quad (63)$$

In *Step 20*, the defuzzified decision values are computed. This value of the QSFS's is calculated using Eq. (56). *Step 21* is about constructing Si, Ri and Qi. First, $\tilde{f}_j$ is obtained. The best $\tilde{f}_j^*$ and worst $\tilde{f}_j^-$ for criteria are computed with Eq. (64) (Hu and Yiu 2024).

$$\tilde{f}_j^* = \max_i \tilde{x}_{ij}, \text{ and } \tilde{f}_j^- = \min_i \tilde{x}_{ij}, \quad (64)$$

The mean group utility and maximal regret are obtained using Eqs. (65) and (66) (Razzaque et al. 2024).

$$\tilde{S}_i = \sum_{j=1}^n \tilde{w}_j \frac{(|\tilde{f}_j^* - \tilde{x}_{ij}|)}{(|\tilde{f}_j^* - \tilde{f}_j^-|)} \quad (65)$$

$$\tilde{R}_i = \max_j \left[\tilde{w}_j \frac{(|\tilde{f}_j^* - \tilde{x}_{ij}|)}{(|\tilde{f}_j^* - \tilde{f}_j^-|)} \right] \quad (66)$$

The final scores of alternatives are calculated with the help of Eq. (67).

$$\tilde{Q}_i = v(\tilde{S}_i - \tilde{S}^*)/(\tilde{S}^- - \tilde{S}^*) + (1 - v)(\tilde{R}_i - \tilde{R}^*)/(\tilde{R}^- - \tilde{R}^*) \quad (67)$$

The maximum group utility approach has a weight of $v=0.5$. The sum of one's personal regrets is $(1 - v)$. Every case is taken into account. After the values of S, R, and Q are sorted, two requirements must be met in order to determine the final ranking of options. Equation (68) yields the result for Condition 1. The scenario takes into account the alternative that is ranked second based on Q value. The next requirement is that the alternative has to be ranked by S, R, or both (Yue 2024).

$$Q(A^{(2)}) - Q(A^{(1)}) \geq 1/(j - 1) \quad (68)$$

If one of these conditions is not satisfied, a set of compromise solutions is selected. The compromise solutions could consist of alternatives and if only condition 2 is not satisfied, or alternatives $A^{(1)}, A^{(2)}, \dots, A^{(M)}$ if condition 1 is not satisfied. These alternatives are selected based on the close positions of their rankings and calculated by the relation for maximum M.

In *Step 22*, ranking values are compared with sensitivity analysis.

4 Analysis results

The results of the analyzes conducted for Metaverse investment decision-making prioritization are detailed in this section with subtitles.

4.1 Prioritizing the experts with AI-based decision-making method

In our study, we define clusters of experts with similar qualifications and subsequently identify their relative importance using an AI-based expert prioritization methodology. The key specifications are considered including education, experience, salary, and age. These specifications were chosen to capture a holistic view of each expert's profile across different dimensions. Salary, despite varying across different companies and industries, serves as a standardized metric that reflects the market valuation of an expert's skills, experience, and qualifications. Including salary helps in understanding an expert's relative position within their respective fields.

Our aim is to understand and cluster experts based on a holistic set of specifications, including salary, to get clear information about their qualifications and prioritize them accordingly. For instance, the clustering outcome, which led to DM2's separate classification, highlights the distinct nature of DM2's profile in the context of our defined criteria. This does not imply that salary alone determined the clustering but rather that the combination of factors (experience, salary, age, and education) together influenced the result. Our approach aims to balance various aspects of an expert's profile. By incorporating salary alongside education, experience, and age, we ensure that the evaluation is not biased towards any single factor but considers multiple dimensions of expertise.

Our AI-based methodology involves clustering the experts using the k-means algorithm. The optimal number of clusters is determined using the elbow method. Once the clusters are formed, we compute the weights of the experts based on the clusters they belong to. The clustering process assigned DM2 to a separate cluster because, despite having more experience and the same salary as DM1, the combination of DM2's specifications (experience and age) differentiated them from the other experts. This indicates that DM2's profile, in the context of all the chosen criteria, was distinct enough to warrant a separate cluster.

The weight of an expert within a cluster is influenced by the cluster's size and the mean standard deviation of features within that cluster. In the case of DM2, the separate cluster assignment resulted in a cluster weight that did not contribute significantly to the overall prioritization process. Consequently, DM2's weight was calculated as 0. The weight of DM2 was found to be 0 because the clustering algorithm identified that DM2 did not fit well into any of the clusters where significant weighting was assigned. This suggests

Table 1 Specifications of experts

Experts	Education	Experience (year)	Salary (USD)	Age
DM1	Bachelor	19	2800	39
DM2	Bachelor	24	2800	44
DM3	PhD	20	3000	46
DM4	Bachelor	22	2750	40
DM5	PhD	20	3250	42
DM6	Master	18	3000	47

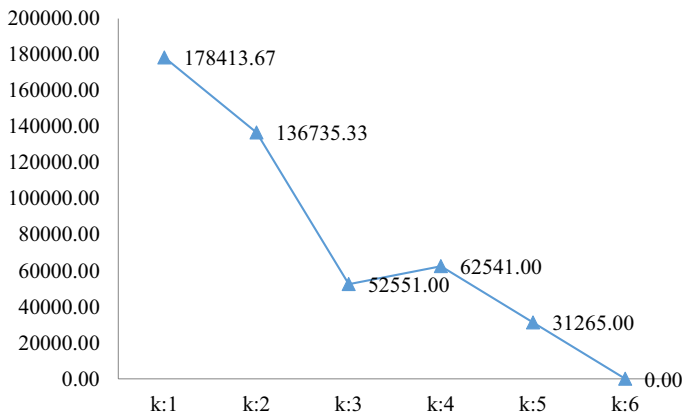


Fig. 2 The plot of the WCSS values

that, according to our AI-based prioritization, DM2's overall profile did not align closely with the other decision makers who were prioritized. So, it is possible to assign the expert groups with similar specifications into the same clusters.

In *Step 1*, the specifications of decision-makers in Table 1 are defined. The decision to incorporate salary alongside other criteria was made to ensure a holistic evaluation of the experts' profiles. Although they originate from various industries, their salaries serve as a common metric for understanding their respective positions within their fields. By considering salary alongside other factors like education, experience, and age, we aim to capture a comprehensive view of their expertise and qualifications, which are crucial for effective decision-making in the metaverse market.

According to Table 1, the experience period of experts varies between 18 and 24 years. Additionally, the average salary of 6 experts is 2933 dollars and the mean of their age is 43. *Step 2* is about computing the optimal k value for clustering the decision makers. WCSS values are calculated for the different value of clusters with Eq. (46). The number of clusters is 5, from 1 to number of experts, the set of WCSS values are shown in Table 10. With the elbow point, the k is determined. Figure 2 shows the graph of WCSS.

According to Fig. 2, k is 3. The end of *Step 3*, the K-means clustering algorithm is employed to cluster experts. The k is applied for defining the clusters of experts using Eqs. (47) and (48). In this case, the k is taken as 3. The iteration outcomes for three distinct clusters are presented in Table 11. In this Table 11, the cluster assignments align with the

Table 2 The weight of the experts with Pareto principle

Decision Makers	Weights	Normalized weights with Pareto principle
DM1	0.06	0.10
DM2	0.00	0.00
DM3	0.29	0.27
DM4	0.06	0.10
DM5	0.29	0.27
DM6	0.29	0.27

Table 3 Criteria set for balanced scorecard-based investment decisions and Investment decision-making priorities in the metaverse market

Criteria	Codes
Finance	CR1
Customer	CR2
Organization	CR3
Learning and growth	CR4
Alternatives	Codes
Regulatory framework	A1
Technology infrastructure	A2
Market conditions	A3
Liquidity changes	A4

initial cluster centers and the mean of elements for the third iteration. Consequently, the expert cluster is determined as follows: DM1 and DM4 are categorized within cluster 1, DM2 is assigned to cluster 2, and DM3, DM5, and DM6 belong to cluster 3. In *Step 4*, the weights of the experts by considering the cluster weights of the experts are computed using Eqs. (49)–(51). The results are presented in Table 12. Using Eqs. (52) and (53), the weights of the experts with pareto principle are calculated. The weights are given in Table 2. According to the results, we have defined three clusters for the decision makers. Table 2 indicates that DM1 and DM4 are grouped together in one cluster, while DM3, DM5, and DM6 form another cluster, sharing the same weighting results. However, DM2 is assigned to a third cluster separately, without any assigned weight. This suggests that DM2 should be omitted from the decision maker groups due to the absence of weighting results.

Table 2 reveals that DM3, DM5, and DM6 exhibit the highest priorities, each having a value of 0.27. The normalized weights of experts are also computed with the Pareto principle to explore all impacts of the decision makers together. The Pareto principle, also known as the 80/20 rule, is a powerful concept used in various fields, including decision-making and resource allocation. It states that for many phenomena, about 80% of the consequences are produced by 20% of the causes. Accordingly, the Pareto principle helps identify the most significant factors in a set of data. For instance, it can help determine which experts (the 20%) are contributing to the majority (80%) of the expert choices. When it comes to computing the relative importance of the experts among them, the Pareto principle can be particularly useful. The normalized weights will be properly considered for weighting and ranking the factors in the following stages. In Table 2, it is seen that DM1, DM3, DM4, DM5 and DM6 have the priorities in the expert team. So, the evaluations of these 5 decision makers except DM2 are considered only to assess the criteria and alternatives.

4.2 Estimate the missing evaluations for the investment decision-making priorities in the metaverse market with expert recommendation system

Step 5 is about determining the criteria and the alternatives for the investment decision-making priorities in the metaverse market. List and codes of alternatives and criteria are presented in Table 3.

The financial performance of a business is an important factor in driving the success of Metaverse investments. In this context, it is appropriate for the business to take some actions to achieve this goal. Operational efficiency is of key importance in reducing the

Table 4 Iterative completion of missing expressions for the criteria

	DM 1	DM 2	DM 3	DM 4	DM 5
CR1–CR2	3	3	4	3 (Iteration 1)	4 (Iteration 2)
CR1–CR3	5	5 (Iteration 1)	5 (Iteration 1)	4	2
CR1–CR4	3	5	3 (Iteration 1)	5	4
CR2–CR1	3	3 (Iteration 1)	3	4	4 (Iteration 1)
CR2–CR3	5 (Iteration 1)	5	5	5 (Iteration 1)	5
CR2–CR4	4 (Iteration 1)	4	5	4	4 (Iteration 1)
CR3–CR1	2	2	2 (Iteration 1)	4	5
CR3–CR2	4	4 (Iteration 1)	4	4 (Iteration 2)	4
CR3–CR4	5	5 (Iteration 1)	5	4	3
CR4–CR1	4 (Iteration 1)	4	4 (Iteration 2)	3	4
CR4–CR2	4 (Iteration 1)	4	4 (Iteration 2)	5	5
CR4–CR3	4 (Iteration 2)	5 (Iteration 2)	4	5	4

Table 5 Iterative completion of missing expressions for the alternatives

	DM 1	DM 2	DM 3	DM 4	DM 5
CR1–A1	4 (Iteration 1)	4	4	3	2
CR1–A2	5	5 (Iteration 1)	5	5 (Iteration 1)	5
CR1–A3	3 (Iteration 1)	3	3	3 (Iteration 1)	3
CR1–A4	4	4	4 (Iteration 2)	5	5 (Iteration 1)
CR2–A1	4 (Iteration 2)	4	5 (Iteration 1)	5	5
CR2–A2	5	5	4	3	3 (Iteration 1)
CR2–A3	3 (Iteration 1)	4	3	4 (Iteration 1)	4
CR2–A4	4 (Iteration 2)	4	5 (Iteration 1)	4	5
CR3–A1	4	5	4	3 (Iteration 1)	3
CR3–A2	5 (Iteration 1)	4	5	4 (Iteration 1)	4
CR3–A3	4	5	5 (Iteration 1)	5	5
CR3–A4	5	5 (Iteration 1)	5	4	4 (Iteration 1)
CR4–A1	3 (Iteration 2)	3	4 (Iteration 1)	5	4
CR4–A2	4	5 (Iteration 1)	5	4	4
CR4–A3	4 (Iteration 1)	4	4	5	5 (Iteration 1)
CR4–A4	3	4 (Iteration 1)	4	5 (Iteration 1)	5

costs of businesses. This situation increases the profitability of the business. Thus, it is possible for Metaverse investments to be more successful. Similarly, customer satisfaction is also crucial in improving the performance of Metaverse investments. To achieve this goal, user interfaces must first be easy. In this process, customer expectations must be considered in the design of applications. Additionally, establishing a technical support unit can increase users' satisfaction by providing quick solutions to their problems. Organizational

effectiveness also plays an important role in the success of Metaverse investments. In this process, team collaboration within the business needs to be increased. Owing to strengthening communication, operational processes within the business will be carried out more successfully. Moreover, learning and development is also an important factor in improving the performance of Metaverse investments. In this context, the knowledge level of employees should be increased through continuous training programs.

In *Step 6*, the linguistic evaluations for the dataset are collected. Linguistic scales and quantum picture fuzzy numbers for evaluation is given in Table 13. Then, linguistic evaluations of experts for the relation matrix are created. The results for M-SWARA are shown in Table 14. The results for VIKOR are illustrated in Table 15. In *Step 7*, the similarity degrees of experts are calculated using Eqs. (54) and (55). For evaluations of criteria, similarity index matrix is shown in Table 16. For evaluations of alternatives, the matrix is given in Table 17. *Step 8* involves computing the unidentified expressions iteratively. Iterative completion of missing expressions for the criteria is shared in Table 4.

Moreover, iterative completion of missing expressions for the alternatives is presented in Table 5.

Table 6 Completed linguistic evaluations of decision makers for the criteria

	CR1	CR2	CR3	CR4
DM1				
CR1		M	VH	M
CR2	M		VH	H
CR3	S	H		VH
CR4	H	H	H	
DM3				
CR1		M	VH	VH
CR2	M		VH	H
CR3	S	H		VH
CR4	H	H	VH	
DM4				
CR1		H	VH	M
CR2	M		VH	VH
CR3	S	H		VH
CR4	H	H	H	
DM5				
CR1		M	H	VH
CR2	H		VH	H
CR3	H	H		H
CR4	M	VH	VH	
DM6				
CR1		H	S	H
CR2	H		VH	H
CR3	VH	H		M
CR4	H	VH	H	

Table 7 Stable matrix

	CR1	CR2	CR3	CR4
CR1	0.216	0.216	0.216	0.216
CR2	0.214	0.214	0.214	0.214
CR3	0.301	0.301	0.301	0.301
CR4	0.269	0.269	0.269	0.269

4.3 Weighting the criteria for the balanced scorecard-based investment decisions using QPFR-M-SWARA

Step 9 involves determining the criteria for the balanced scorecard-based investment decisions. The four criteria given in Table 3 are created on the principle of the balance score card. In *Step 10*, the completed linguistic evaluations of decision makers for the criteria are constructed. The evaluations are given in Table 6.

In the end of *Step 11*, the QPFRN for the relation matrix are defined. The results obtained with Eq. (56) are depicted in Table 18. In *Step 12*, using Eqs. (57) and (58), the weighted values given in Table 19 are obtained. In *Step 13*, defuzzified values are computed with Eq. (59). The defuzzified values are given in Table 20. In *Step 14*, the normalized relationship matrix is found. The normalized relationship matrix is shown in Table 21. Then, *Step 15* is about the calculation of s_j , k_j , q_j and w_j values for the relationship value of each criterion in Eqs. (60)–(62). The values are detailed in Table 22. *Step 16* involves constructing the relationship matrix and the direction among the criteria. The result details of the analysis are given in Table 23. A stable matrix is calculated with a sufficiently large t value. Stable matrix is tabulated in Table 7.

According to Table 7, the most influential element in metaverse investment decisions is Organization. The weight of this criterion is 0.301. As a result of M-SWARA analysis, the second effective criterion is learning and growth. customer is at the last place in the

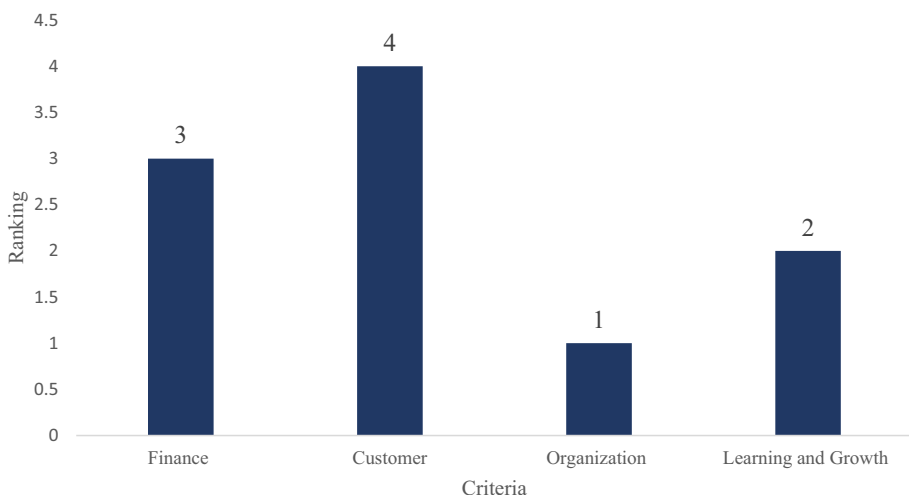
**Fig. 3** Ranking of criteria in investment decision

Table 8 Completed linguistic evaluations of decision makers for the alternatives

	CR1	CR2	CR3	CR4
DM1				
A1	G	G	G	F
A2	B	B	B	G
A3	F	F	G	G
A4	G	G	G	F
DM3				
A1	G	G	B	F
A2	B	B	G	B
A3	F	G	B	G
A4	G	G	B	G
DM4				
A1	G	B	G	G
A2	B	G	B	B
A3	F	F	B	G
A4	G	B	B	G
DM5				
A1	F	B	F	B
A2	B	F	G	G
A3	F	G	B	B
A4	B	G	G	B
DM6				
A1	P	B	F	G
A2	B	F	G	G
A3	F	G	B	B
A4	B	B	G	B

Table 9 Comparative ranking values with sensitivity analysis

Alternatives	Case 1	Case 2	Case 3	Case 4
Extended VIKOR (v:0.5)				
A1	1	1	1	1
A2	4	4	3	3
A3	3	3	4	4
A4	2	2	2	2
Extended TOPSIS				
A1	1	1	1	1
A2	4	4	3	3
A3	3	3	4	4
A4	2	2	2	2

ranking. The importance weight of this criterion is calculated as 0.214. The ranking of the criteria is visualized in Fig. 3.

4.4 Ranking the investment decision-making priorities in the metaverse market with QPFR-VIKOR

In *Step 17*, the completed linguistic evaluations of decision makers for the alternatives are constructed. The linguistic evaluations are given in Table 8.

Step 18 is about constructing the QPFRN for the X. The matrix is shown in Table 24. In *Step 19*, the expert weighted QPFRS for X is determined in Table 25. *Step 20* involves computing defuzzified values using Eq. (56). The defuzzified values are shown in Table 26. At the end of *Step 21*, S, R and Q values are constructed with the help of Eqs. (64)–(68). Q values calculated for 10 different types of v with S and R values are presented in Table 27. In case of maximum group utility, veto, and consensus except v:1, Regulatory Framework demonstrates superior ranking performance compared to the other alternatives. In *Step 22*, the comparative ranking values with sensitivity analysis are calculated. TOPSIS and VIKOR analysis results are reported in Table 9.

In two methods and 8 cases, the regulatory framework ranked first. The second-best alternative is calculated as liquidity changes. In other words, it can be said that the analysis results obtained are consistent. Market conditions and technology infrastructure vary depending on the situation and method. However, it does not change the order of the other criteria and is ranked last.

5 Discussions

Organizational effectiveness is found to be the most important factor in improving the performance of metaverse investments. In this context, different departments in the business must work in harmony with each other. This situation allows more accurate strategies to be developed to increase the effectiveness of metaverse investments. Thanks to different departments working in harmony with each other, it can be easier to optimize business processes. On the other hand, Queiroz et al. (2023) demonstrated that Metaverse projects often require advanced technological infrastructures. By ensuring organizational effectiveness, it becomes possible for businesses to develop this infrastructure. Similarly, businesses can adapt to new developing technologies more easily. Moreover, Metaverse projects require collaboration between different departments and teams. Khaliq and Manda (2024) identified that by ensuring organizational effectiveness, cooperation within the business can be increased. Thus, more successful metaverse investments can be made. In this context, investors need to take some actions to increase organizational effectiveness. To achieve this goal, employees must be given the necessary training on metaverse technologies. Volchek and Brysch (2023) denoted that this issue contributes significantly to ensuring organizational effectiveness. Moreover, an innovation culture needs to be developed within the business. Chan et al. (2023) showed that when the working environment at the workplace supports innovation, it becomes easier to increase communication between departments.

It is concluded that it is important for businesses to ensure technological improvements in the development of Metaverse investments. Since Metaverse projects involve comprehensive processes, the integration of advanced technologies is required. Therefore, by using innovative technologies, businesses can gain a significant competitive advantage over their competitors. Similarly, Buhalis et al. (2023) stated that the use of high technology also supports customer satisfaction. With the advanced technological infrastructure, the expectations of customers trading on the platform can be met more easily. On the other hand, Ning et al. (2023) mentioned that digital asset trading takes place on the Metaverse platform. Advanced technologies are needed to trade these assets more successfully. Moreover, the technological infrastructure needs to be developed to ensure the security of transactions within the platform. Considering these issues, businesses need to make some improvements to ensure technological development. Kou et al. (2023a, b) discussed that it is important to make correct budgeting to ensure high performance of technological investments on the Metaverse platform. In addition to them, Kraus et al. (2023) indicated that up-to-date hardware and software must be used to support Metaverse applications. This application allows disruptions in operational processes to be minimized.

6 Conclusion

In this study, it is aimed to examine the main indicators of the performance of the metaverse investments. Within this context, a novel model is created that has four different stages. The first stage is related to the prioritizing the experts with artificial intelligence-based decision-making method. Secondly, missing evaluations are estimated by expert recommendation system. Thirdly, the criteria are weighted with QPFR M-SWARA. Finally, investment decision-making priorities are ranked by QPFR VIKOR. Organizational effectiveness is found to be the most important factor in improving the performance of metaverse investments. Similarly, it is also identified that it is important for businesses to ensure technological improvements in the development of Metaverse investments. On the other side, the ranking results indicate that regulatory framework is the most critical alternative for this situation.

The main contribution of this study is the integration of the artificial intelligence methodology to the fuzzy decision-making approach to calculate the weights of the decision makers. With the help of this issue, the evaluations of these people are examined according to their qualifications. This situation has a positive contribution to make more appropriate examinations. Technological advances are key to increasing the effectiveness of Metaverse investments. It is possible to offer some policy suggestions to achieve this goal. First of all, governments can provide tax breaks to encourage research and development activities for Metaverse technologies. These supports will provide significant cost advantages to businesses. It is of critical importance in this context for the development of education. In particular, it may be possible to obtain a more qualified workforce through training on technology and software development. Legal regulations are another issue that should be taken into consideration in this process. Adequacy of these regulations contributes to increasing users' confidence in the market.

The main limitation of the proposed model is that details of work projects carried out cannot be taken into consideration while weighting the decision makers. Thus, in the future

studies, new decision-making models can be created to satisfy this situation. Selecting the criteria by considering balanced scorecard perspectives is another theoretical limitation. Different approaches can be taken into consideration to select the performance indicators in the following studies, such as activity-based costing and customer lifetime value. On the other side, VIKOR methodology is considered to rank the alternatives. This approach is criticized in the literature due to some situations. Therefore, as a future research direction, a new ranking model can be generated to overcome all criticisms regarding the existing ranking techniques. Another significant limitation of the proposed model is related to the picture fuzzy sets. For these sets to be used effectively, sufficient data is needed to explain the abstention situation. This situation creates some difficulties in the analysis process.

Appendix

See Tables [10](#), [11](#), [12](#), [13](#), [14](#), [15](#), [16](#), [17](#), [18](#), [19](#), [20](#), [21](#), [22](#), [23](#), [24](#), [25](#), [26](#), [27](#).t

Table 10 The set of WCSS values for different k values

K = 1	K = 2		K = 3		K = 4		K = 5		K = 6	
	WCSS	WCSS	WCSS	WCSS	WCSS	WCSS	WCSS	WCSS	WCSS	WCSS
Cluster 1		Cluster 1	Cluster 1	Cluster 1	Cluster 1	Cluster 1	Cluster 1	Cluster 1	Cluster 1	Cluster 1
DM1	17,796.72	DM1	DM1	DM1	DM1	DM1	DM1	DM1	DM1	DM1
DM2	17,791.72	DM4	DM6	DM6	Cluster 2	Cluster 2	Cluster 2	Cluster 2	Cluster 2	Cluster 2
DM3	4455.06	DM6	Cluster 2	Cluster 2	DM2	DM2	DM2	DM2	DM2	DM2
DM4	33,623.06	Cluster 2	DM2	DM2	Cluster 3	Cluster 3	Cluster 3	Cluster 3	Cluster 3	Cluster 3
DM5	100,280.39	DM2	DM4	DM4	DM3	DM3	DM3	DM3	DM3	DM3
DM6	4466.72	DM3	Cluster 3	Cluster 3	DM5	DM5	Cluster 4	Cluster 4	Cluster 4	Cluster 4
		DM5	DM3	DM5	Cluster 4	Cluster 4	DM4	DM4	DM4	DM4
			DM5	DM5	DM4	DM4	Cluster 5	Cluster 5	Cluster 5	Cluster 5
					DM6	DM6	DM5	DM5	DM5	DM5
					Total	Total	Total	Total	Total	Total
Total	178,413.67	Total	Total	Total	52,551.00	62,541.00	15,632.50	15,632.50	31,265.00	0

Table 11 Iteration results of optimal cluster value

Iteration 1 (DM 1 is in cluster 1; DM2 is in cluster 2; DM 3 is in cluster 3)

Decision maker	Distance to C1	Distance to C2	Distance to C3	Cluster assignment
Initial cluster centers				
DM1	0.00	7.07	200.13	1
DM2	7.07	0.00	200.06	2
DM3	200.13	200.06	0.00	3
DM4	50.10	50.20	250.09	1
DM5	450.02	450.03	250.03	3
DM6	200.16	200.11	2.45	3
Average of data points				
DM1	25.05	7.07	283.40	2
DM2	25.64	0.00	283.38	2
DM3	225.10	200.06	83.34	3
DM4	25.05	50.20	333.39	1
DM5	475.01	450.03	166.70	3
DM6	225.14	200.11	83.37	3

Iteration 2 (DM 1 is in cluster 1; DM2 is in cluster 2; DM 4 is in cluster 3)

Decision maker	Distance to C1	Distance to C2	Distance to C3	Cluster assignment
Initial cluster centers				
DM1	0.00	7.07	50.10	1
DM2	7.07	0.00	50.20	2
DM3	200.13	200.06	250.09	2
DM4	50.10	50.20	0.00	3
DM5	450.02	450.03	500.01	1
DM6	200.16	200.11	250.13	2
Average of data points				
DM1	225.01	133.51	50.10	3
DM2	225.07	133.39	50.20	3
DM3	25.62	66.68	250.09	1
DM4	275.01	183.43	0.00	3
DM5	225.01	316.69	500.01	1
DM6	25.87	66.73	250.13	1

Iteration 3 (DM 1 is in cluster 1; DM2 is in cluster 2; DM 5 is in cluster 3)

Decision maker	Distance to C1	Distance to C2	Distance to C3	Cluster Assignment
Initial cluster centers				
DM1	0.00	7.07	450.02	1
DM2	7.07	0.00	450.03	2
DM3	200.13	200.06	250.03	3
DM4	50.10	50.20	500.01	1
DM5	450.02	450.03	0.00	3

Table 11 (continued)

Iteration 3 (DM 1 is in cluster 1; DM2 is in cluster 2; DM 5 is in cluster 3)

Decision maker	Distance to C1	Distance to C2	Distance to C3	Cluster Assignment
DM6	200.16	200.11	250.06	3
Average of data points				
DM1	7.07	25.05	283.40	1
DM2	25.64	0.00	283.38	2
DM3	225.10	200.06	83.34	3
DM4	25.05	50.20	333.39	1
DM5	475.01	450.03	166.70	3
DM6	225.14	200.11	83.37	3

Table 12 The standard deviations of the features and the weights by clusters

Cluster center	Size	Institutions	Duty	Experience	Graduation	Mean SD	Weight
C1	2	0	1.5	25	0.5	6.75	13.5
C2	1	0.00	0.00	0.00	0.00	0.00	0
C3	3	0.47	0.94	117.85	2.16	30.36	91.07

Table 13 Linguistic scales and quantum picture fuzzy numbers for evaluation

Linguistic scales for criteria	Linguistic scales for alternatives	Recommendation degrees	Possibility degrees	QPFNs
No influence (n)	Weakest (w)	1	0.40	$\left[\begin{array}{l} \sqrt{0.16}e^{j2\pi \cdot 0.4}, \\ \sqrt{0.10}e^{j2\pi \cdot 0.25}, \\ \sqrt{0.46}e^{j2\pi \cdot 0.22}, \\ \sqrt{0.28}e^{j2\pi \cdot 0.13} \end{array} \right]$
Somewhat influence (s)	Poor (p)	2	0.45	$\left[\begin{array}{l} \sqrt{0.20}e^{j2\pi \cdot 0.45}, \\ \sqrt{0.13}e^{j2\pi \cdot 0.28}, \\ \sqrt{0.42}e^{j2\pi \cdot 0.17}, \\ \sqrt{0.25}e^{j2\pi \cdot 0.10} \end{array} \right]$
Medium influence (m)	Fair (f)	3	0.50	$\left[\begin{array}{l} \sqrt{0.25}e^{j2\pi \cdot 0.50}, \\ \sqrt{0.15}e^{j2\pi \cdot 0.31}, \\ \sqrt{0.37}e^{j2\pi \cdot 0.12}, \\ \sqrt{0.23}e^{j2\pi \cdot 0.07} \end{array} \right]$
High influence (h)	Good (g)	4	0.55	$\left[\begin{array}{l} \sqrt{0.30}e^{j2\pi \cdot 0.55}, \\ \sqrt{0.19}e^{j2\pi \cdot 0.34}, \\ \sqrt{0.32}e^{j2\pi \cdot 0.07}, \\ \sqrt{0.19}e^{j2\pi \cdot 0.04} \end{array} \right]$
Very high influence (vh)	Best (b)	5	0.60	$\left[\begin{array}{l} \sqrt{0.36}e^{j2\pi \cdot 0.6}, \\ \sqrt{0.22}e^{j2\pi \cdot 0.37}, \\ \sqrt{0.26}e^{j2\pi \cdot 0.02}, \\ \sqrt{0.16}e^{j2\pi \cdot 0.01} \end{array} \right]$

Table 14 Linguistic evaluations of the decision makers for the relation matrix

	DM 1	DM 2	DM 3	DM 4	DM 5
CR1–CR2	M	M	H	n/a	n/a
CR1–CR3	VH	n/a	n/a	H	S
CR1–CR4	M	VH	n/a	VH	H
CR2–CR1	M	n/a	M	H	n/a
CR2–CR3	n/a	VH	VH	n/a	VH
CR2–CR4	n/a	H	VH	H	n/a
CR3–CR1	S	S	n/a	H	VH
CR3–CR2	H	n/a	H	n/a	H
CR3–CR4	VH	n/a	VH	H	M
CR4–CR1	n/a	H	n/a	M	H
CR4–CR2	n/a	H	n/a	VH	VH
CR4–CR3	n/a	n/a	H	VH	H

Table 15 Linguistic evaluations of the decision makers for the decision matrix

	DM 1	DM 2	DM 3	DM 4	DM 5
CR1–A1	n/a	G	G	F	P
CR1–A2	B	n/a	B	n/a	B
CR1–A3	n/a	F	F	n/a	F
CR1–A4	G	G	n/a	B	n/a
CR2–A1	n/a	G	n/a	B	B
CR2–A2	B	B	G	F	n/a
CR2–A3	n/a	G	F	n/a	G
CR2–A4	n/a	G	n/a	G	B
CR3–A1	G	B	G	n/a	F
CR3–A2	n/a	G	B	n/a	G
CR3–A3	G	B	n/a	B	B
CR3–A4	B	n/a	B	G	n/a
CR4–A1	n/a	F	n/a	B	G
CR4–A2	G	n/a	B	G	G
CR4–A3	n/a	G	G	B	n/a
CR4–A4	F	n/a	G	n/a	B

Table 16 Similarity index matrix of the decision makers for the criteria

	DM 1	DM 2	DM 3	DM 4	DM 5
DM 1	1.00	0.38	0.35	− 0.12	− 0.75
DM 2	0.38	1.00	0.24	0.24	− 0.08
DM 3	0.35	0.24	1.00	− 0.07	0.00
DM 4	− 0.12	0.24	− 0.07	1.00	0.23
DM 5	− 0.75	− 0.08	0.00	0.23	1.00

Table 17 Similarity index matrix of the decision makers for the alternatives

	DM 1	DM 2	DM 3	DM 4	DM 5
DM 1	1.00	0.06	0.26	− 0.32	− 0.06
DM 2	0.06	1.00	0.19	− 0.25	0.16
DM 3	0.26	0.19	1.00	− 0.02	0.31
DM 4	− 0.32	− 0.25	− 0.02	1.00	0.45
DM 5	− 0.06	0.16	0.31	0.45	1.00

[illegible]

Table 18 (continued)

[illegible]

Table 18 (continued)

	CR1	CR2	CR3	CR4
CR3	$\begin{bmatrix} \sqrt{0.36}e^{j2\pi \cdot 0.6}, \\ \sqrt{0.22}e^{j2\pi \cdot 0.37}, \\ \sqrt{0.26}e^{j2\pi \cdot 0.02}, \\ \sqrt{0.16}e^{j2\pi \cdot 0.01} \end{bmatrix}$	$\begin{bmatrix} \sqrt{0.30}e^{j2\pi \cdot 0.55}, \\ \sqrt{0.19}e^{j2\pi \cdot 0.34}, \\ \sqrt{0.32}e^{j2\pi \cdot 0.07}, \\ \sqrt{0.19}e^{j2\pi \cdot 0.04} \end{bmatrix}$		$\begin{bmatrix} \sqrt{0.25}e^{j2\pi \cdot 0.50}, \\ \sqrt{0.15}e^{j2\pi \cdot 0.31}, \\ \sqrt{0.37}e^{j2\pi \cdot 0.12}, \\ \sqrt{0.23}e^{j2\pi \cdot 0.07} \end{bmatrix}$
CR4	$\begin{bmatrix} \sqrt{0.30}e^{j2\pi \cdot 0.55}, \\ \sqrt{0.19}e^{j2\pi \cdot 0.34}, \\ \sqrt{0.32}e^{j2\pi \cdot 0.07}, \\ \sqrt{0.19}e^{j2\pi \cdot 0.04} \end{bmatrix}$	$\begin{bmatrix} \sqrt{0.36}e^{j2\pi \cdot 0.6}, \\ \sqrt{0.22}e^{j2\pi \cdot 0.37}, \\ \sqrt{0.26}e^{j2\pi \cdot 0.02}, \\ \sqrt{0.16}e^{j2\pi \cdot 0.01} \end{bmatrix}$	$\begin{bmatrix} \sqrt{0.30}e^{j2\pi \cdot 0.55}, \\ \sqrt{0.19}e^{j2\pi \cdot 0.34}, \\ \sqrt{0.32}e^{j2\pi \cdot 0.07}, \\ \sqrt{0.19}e^{j2\pi \cdot 0.04} \end{bmatrix}$	

[illegible]

Table 20 The defuzzified values of QPFRSs

	CR1	CR2	CR3	CR4
CR1	0.000	0.060	0.071	0.066
CR2	0.061	0.000	0.069	0.062
CR3	0.065	0.062	0.000	0.070
CR4	0.062	0.068	0.068	0.000

Table 21 The normalized relation matrix

	CR1	CR2	CR3	CR4
CR1	0.000	0.306	0.360	0.333
CR2	0.316	0.000	0.360	0.323
CR3	0.330	0.314	0.000	0.356
CR4	0.316	0.342	0.342	0.000

Table 22 Sj, kj, qj, and wj values for the relationship degrees of each criterion

CR1	Sj	kj	qj	Wj	CR2	Sj	Kj	qj	Wj
CR3	0.360	1.000	1.000	0.430	CR3	0.360	1.000	1.000	0.429
CR4	0.333	1.333	0.750	0.323	CR4	0.323	1.323	0.756	0.324
CR2	0.306	1.306	0.574	0.247	CR1	0.316	1.316	0.574	0.246
CR3	Sj	kj	qj	wj	CR4	Sj	Kj	qj	Wj
CR4	0.356	1.000	1.000	0.430	CR3	0.342	1.000	1.000	0.433
CR1	0.330	1.330	0.752	0.323	CR2	0.342	1.342	0.745	0.322
CR2	0.314	1.314	0.572	0.246	CR1	0.316	1.316	0.566	0.245

Table 23 Relation matrix with the values of wj and the impact directions

	CR1	CR2	CR3	CR4	Impact directions
CR1		0.247	0.430	0.323	C1 → C3
CR2	0.246		0.429	0.324	C2 → C3
CR3	0.323	0.246		0.430	C3 → C4
CR4	0.245	0.322	0.433		C4 → C3

[illegible]

[illegible]

Table 24 (continued)

	CR1	CR2	CR3	CR4
A3	$\begin{bmatrix} \sqrt{0.25}e^{j2\pi \cdot 0.50}, \\ \sqrt{0.15}e^{j2\pi \cdot 0.31}, \\ \sqrt{0.37}e^{j2\pi \cdot 0.12}, \\ \sqrt{0.23}e^{j2\pi \cdot 0.07} \end{bmatrix}$	$\begin{bmatrix} \sqrt{0.30}e^{j2\pi \cdot 0.55}, \\ \sqrt{0.19}e^{j2\pi \cdot 0.34}, \\ \sqrt{0.32}e^{j2\pi \cdot 0.07}, \\ \sqrt{0.19}e^{j2\pi \cdot 0.04} \end{bmatrix}$	$\begin{bmatrix} \sqrt{0.36}e^{j2\pi \cdot 0.6}, \\ \sqrt{0.22}e^{j2\pi \cdot 0.37}, \\ \sqrt{0.26}e^{j2\pi \cdot 0.02}, \\ \sqrt{0.16}e^{j2\pi \cdot 0.01} \end{bmatrix}$	$\begin{bmatrix} \sqrt{0.36}e^{j2\pi \cdot 0.6}, \\ \sqrt{0.22}e^{j2\pi \cdot 0.37}, \\ \sqrt{0.26}e^{j2\pi \cdot 0.02}, \\ \sqrt{0.16}e^{j2\pi \cdot 0.01} \end{bmatrix}$
A4	$\begin{bmatrix} \sqrt{0.36}e^{j2\pi \cdot 0.6}, \\ \sqrt{0.22}e^{j2\pi \cdot 0.37}, \\ \sqrt{0.26}e^{j2\pi \cdot 0.02}, \\ \sqrt{0.16}e^{j2\pi \cdot 0.01} \end{bmatrix}$	$\begin{bmatrix} \sqrt{0.36}e^{j2\pi \cdot 0.6}, \\ \sqrt{0.22}e^{j2\pi \cdot 0.37}, \\ \sqrt{0.26}e^{j2\pi \cdot 0.02}, \\ \sqrt{0.16}e^{j2\pi \cdot 0.01} \end{bmatrix}$	$\begin{bmatrix} \sqrt{0.30}e^{j2\pi \cdot 0.55}, \\ \sqrt{0.19}e^{j2\pi \cdot 0.34}, \\ \sqrt{0.32}e^{j2\pi \cdot 0.07}, \\ \sqrt{0.19}e^{j2\pi \cdot 0.04} \end{bmatrix}$	$\begin{bmatrix} \sqrt{0.36}e^{j2\pi \cdot 0.6}, \\ \sqrt{0.22}e^{j2\pi \cdot 0.37}, \\ \sqrt{0.26}e^{j2\pi \cdot 0.02}, \\ \sqrt{0.16}e^{j2\pi \cdot 0.01} \end{bmatrix}$

 Springer[illegible]

Table 26 The defuzzified decision values

Criteria/alternatives	CR1	CR2	CR3	CR4
A1	0.063	0.068	0.068	0.066
A2	0.069	0.068	0.064	0.068
A3	0.055	0.060	0.067	0.068
A4	0.068	0.068	0.068	0.066

Table 27 S, R and Q values

	Si	Ri	Qi (v:0.1)	Qi (v:0.2)	Qi (v:0.3)	Qi (v:0.4)
A1	0.324	0.220	0.049	0.048	0.047	0.046
A2	0.315	0.301	0.900	0.800	0.700	0.600
A3	0.529	0.216	0.100	0.200	0.300	0.400
A4	0.346	0.269	0.569	0.522	0.475	0.428
	Qi (v:0.5)	Qi (v:0.6)	Qi (v:0.7)	Qi (v:0.8)	Qi (v:0.9)	Qi (v:1)
A1	0.045	0.044	0.042	0.041	0.040	0.039
A2	0.550	0.400	0.300	0.200	0.100	0.000
A3	0.500	0.600	0.700	0.800	0.900	1.000
A4	0.380	0.333	0.286	0.239	0.191	0.144

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Declarations

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